Order of the differential equation : order of highest derivative

e.g $\frac{dx}{dt}$ ..... = 1^{st} order

$\frac{dx}{dy}$ .... = 1^{st} order

$\frac{d^2y}{dx^2}$ .... = 2^{nd} order

$\frac{dy}{dx} + \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}$ ... = 3^{rd} order etc.

- the order tells us the number of constants of integration
  ↳ number of initial conditions needed to find solution

All differential operators count as linear terms with respect to the dependent variable

$$\underbrace{\frac{dx}{dt} \cdot \frac{d^2x}{dt^2} \quad \frac{d^3x}{dt^3}}_{\text{all linear in } x}$$

Linearity : important as linear equations easier to solve

ODE linear if dependant variables & its derivatives do not appear as products, raised to powers, or as part of non-linear functions
↳ trig, exp. etc

e.g

$\frac{dy}{dx} + 5y = \cos t$ → linear (cos w/r to t not the dependant variable, x)

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - y = 0$ → linear

$\frac{dy}{dt} + 5y + \cos y = 0$ → non-linear (cos w/r to dependent y)

$\frac{dy}{dt} - 6y\frac{dy}{dt} = 2t$ → non linear ($6y \times \frac{dy}{dt}$ = linear × linear = non linear)

$\frac{d^2y}{dt^2} + 2t^3\frac{dy}{dx} = 5ty + t^2$ → linear (although t is non-linear, y is the dependent variable)

Homogenity : we write ODEs with all terms involving the dependent variable on the left side.

- Homogenous if right side is 0
- Non-homogenous otherwise

e.g

$\frac{dy}{dt} + 5y = \cos t \longrightarrow$ linear, non-homogenous

$2y\frac{dy}{dt} - \frac{d^2y}{dt^2} - y = 0 \longrightarrow$ non-linear, homogenous